JH ASTRONOMY AND ASTROPHYSICS : AS 3015

Nebulae: Tutorial Questions 3

1. Neutrinos have an equilibrium intensity given by,

$$I_\nu = \frac{2h\nu^3/c^2}{\exp(h\nu/kT)+1}$$

Assume that the neutrinos are in thermal equilibrium with atoms (not normally the case!) and use the Einstein relations to derive the transition rate between two atomic energy levels (ie the neutrino equivalent of $n_1 B_{12}\bar{J} = n_2 A_{21} + n_2 B_{21}\bar{J}$).
Can you explain why this is different from the photon case?

2. If the population of state i in a multi-level atom can be written as
$N_i = \frac{N}{U} g_i e^{-E_i/kT}$
and the total number of atoms is
$N = \sum N_i$
derive an expression for the partition function U in terms of the statistical weights g_i and level energies E_i above the ground state.

Evaluate the first six terms in your expression for U, using expressions for the energies and statistical weights of levels in a hydrogen-like atom at temperatures of 5000K and 10000K. Comment on the implications.

Compute the population ratio n_2/n_1 for the first two levels of atomic hydrogen at these two temperatures.

What density would the gas need to have in order for the Bohr radius at the n=5 level to be comparable to the mean interatomic distance? Does this seem plausible for a nebular environment?

3. The probability density function for the velocity components of particles in a gas at temperature T is
$dN(\upsilon_x, \upsilon_y, \upsilon_z) \propto e^{-m(\upsilon_x^2+\upsilon_y^2+\upsilon_z^2)/2kT} d\upsilon_x d\upsilon_y d\upsilon_z$.
Show that the total probability enclosed by the volume of (υ_x, υ_y, υ_z) space with speeds in the range υ to $\upsilon+d\upsilon$ is $dN(\upsilon) \propto \upsilon^2 e^{-m\upsilon^2/2kT} d\upsilon.$
(Hint: what sort of surface in (υ_x, υ_y, υ_z) space would particles that all have the same speed lie on, and how does the area of this surface depend on υ ?)
Integrate over all speeds to show that the normalized probability distribution for particle speeds is
$$dN(\upsilon) = \sqrt{\frac{2}{\pi}} \left(\frac{m}{kT}\right)^{3/2} \upsilon^2 e^{-m\upsilon^2/2kT} d\upsilon.$$
Show that the most probable speed is $\upsilon = (2kT/m)^{1/2}$, and that the mean speed is at $\upsilon = (8kT/\pi m)^{1/2}$.