JH ASTRONOMY AND ASTROPHYSICS : AS 3015

Nebulae: Tutorial Questions 4

1. Nebular radiative transfer problems involving scattering in the vicinity of a stellar source of photons often require us to compute the mean intensity J_ν. Show that, for a star of radius R whose photosphere emits isotropically with uniform specific intensity I_ν, the mean intensity at distance r from the star is
$$J_\nu = \frac{I_\nu}{2}\left(1-\sqrt{1-\frac{R^2}{r^2}}\right).$$
Hence show that at the stellar surface $J_\nu = I_\nu / 2$ and that at large distances from the star,
$$J_\nu \to \frac{I_\nu}{4}\frac{R^2}{r^2}.$$

2. Describe the physical picture of the stages in the expansion of an HII region into a uniform medium. Explain how recombinations in the interior of the region affect the ionizing flux at the front.

3. The O^{++} ion has a triplet ^3P term for the ground state with J=0, 1, 2. There is a singlet $^1D_2$ term at energy ΔE~kT above the ground state, and a singlet $^1S_0$ term at energy ΔE~kT above the $^1D_2$ term. Downward transitions from $^1S_0$ to $^1D_2$ emit line photons with wavelength 4363 Å. Downward transitions from $^1D_2$ to $^3P_2$ and $^3P_1$ emit at 5007 Å and 4959 Å respectively. Sketch the energy-level diagram showing the five levels and the three lines. Given that the ratio of the lines' Einstein coefficients is A(λ5007) ~3A(λ4959), predict the observed flux ratio of these two lines in the low-density limit where spontaneous emission occurs faster than collisional de-excitation. Justify your reasoning.

4. A distant HII region in the Milky Way is found to have a Balmer recombination line flux ratio $H\alpha/H\beta = 4.0$. Given that this flux ratio is close to 2.86 for unreddened HII regions, and that the extinction A_λ (in magnitudes) varies inversely with wavelength λ, calculate the de-reddening factors by which the observed $H\alpha$ and $H\beta$ line fluxes must be multiplied to remove the effects of extinction.

5. The electronic energy as a function of the internuclear separation R in a diatomic molecule can be approximated using a potential,
$$E(R) = E_0\left[1-e^{-(R-R_0)/L}\right]^2 - E_0$$
where E_0, R_0 and L are constants. Sketch this potential, and show that it has a minimum at $R = R_0$.

6. In a Monte Carlo code that simulates emission and scattering in a spherical circumstellar shell, the radial dependence of the emissivity is $j(r) \propto (r/R)^{-\alpha}$, where r is in the range $R < r < R_{max}$. Derive an expression for randomly sampling the radial location for emitting photons in the shell.

7. The Rayleigh scattering phase function is independent of azimuthal angle, ϕ, and has the dependence on polar angle, θ: $P(\theta) \propto 1 + \cos^2 \theta$. What is the normalization factor so that the scattering phase function is normalized over all solid angles? How would you choose θ and ϕ values to randomly choose a scattering direction? Hint: you may not be able to derive analytic expressions for randomly choosing both θ and ϕ.