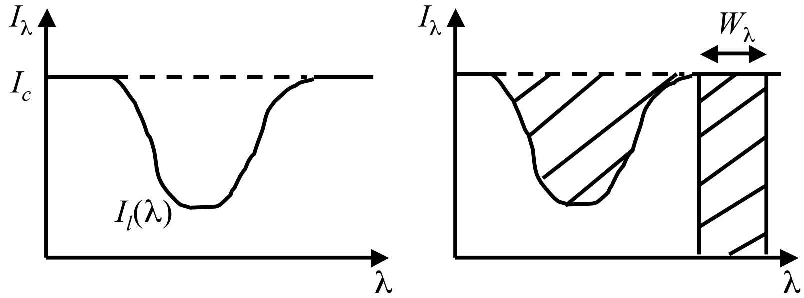
The Line Spectrum

A spectral line is described by its profile. The line depression, D_λ, compares the intensity in the line with the nearby continuum. The equivalent width is the integral of D_λ and is the same width as a rectangular piece of spectrum that blocks the emergent intensity:

$$\boxed{D_\lambda = \frac{I_c - I_\lambda^l}{I_c}}$$

$$\boxed{W_\lambda = \int_{\text{line}} D_\lambda \, d\lambda = \int_{\text{line}} \frac{I_c - I_\lambda^l}{I_c} \, d\lambda}$$

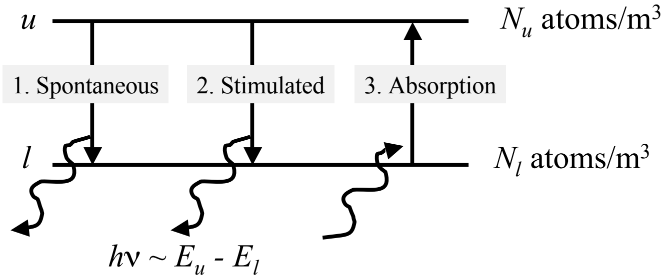


Observationally: Line profiles for weak lines difficult to determine accurately due to distortion caused by finite width of spectrograph slit. Total energy subtracted from continuum is not affected, this measures the integrated line strength or equivalent width.

Theoretically: Line intensity can be calculated from ERT if a_n is known. These depend on atomic transition coefficients, level occupation numbers and the intrinsic line profile for the relevant transition. The profile gives the probability of a photon with n being absorbed or emitted. This depends on the atmospheric temperature and pressure at the particular depth considered. The problem can be broken into two parts: (1) Investigation of intrinsic line profile
(2) Solution of ERT for the line

An important assumption is that line and continuum can be solved separately. If line abs/emis is strong this is not valid and T-structure depends on line transfer and line/cont must be solved simultaneously.

Intrinsic Line Profile



Recall, processes contributing to the line profile are:

1. Spontaneous emission:	Spontaneous de-excitation: A_{ul}
2. Stimulated emission:	Induced de-excitation B_{ul}
3. Absorption:	Radiative excitation: B_{lu}

Number of spontaneous transitions /m^3 /sec = $N_u\, A_{ul}$
Number of stimulated transitions /m^3 /sec = $N_u\, B_{lu}\, J_{\nu 0}$
Number of spontaneous transitions /m^3 /sec = $N_l\, B_{ul}\, J_{\nu 0}$

Line Broadening Mechanisms

Complete redistribution: spontaneous emission, stimulated emission, and absorption line profile shapes are the same. Lines are not sharp due to some or all of the following broadening mechanisms:

- Natural/radiation damping: lifetimes of excited states
- Collisional: collisions/perturbations by other particles
- Doppler: thermal motions
- Rotational: stellar rotation
- Turbulent: mass motions in atmosphere
- Zeeman splitting: magnetic effects

Other Broadening Mechanisms

Rotational Broadening: Stellar rotation, $\Delta\lambda \sim \lambda_0\, v/c \sin i$

Zeeman Splitting: Magnetic fields cause splitting (broadening).

Micro & Macro Turbulence: Mass motions in photosphere. Usually assumed to be Maxwellian in nature, so get:

$$\boxed{\Delta\nu_D \equiv \frac{\nu_0}{c}\sqrt{\frac{2kT}{m} + \xi^2_{\mathrm{micro}}}}$$

Macroturbulence is added by convolving emergent model line profile with a Gaussian velocity distribution ~ $\exp(-\xi^2/\xi^2_{\mathrm{macro}})$ BUT… these are VERY uncertain and are really just fudge factors to get model spectra to match observations.

Residual Flux in a Line

Opacity due to line and continuum: $\boxed{\kappa_\nu = \kappa^c_\nu + \kappa^l_\nu = \kappa^c + \kappa^l_\nu}$

Continuum opacity varies slowly with ν => constant in line

Write: $\boxed{\eta_\nu = \kappa^l_\nu / \kappa^c}$ and assume η_n independent of τ

Now consider total / continuum optical depth:

$$\boxed{\begin{aligned} \mathrm{d}\tau_\nu &= -\rho\,(\kappa^c + \kappa^l_\nu)\,\mathrm{d}z \\ \mathrm{d}\tau &= -\rho\,\kappa^c\,\mathrm{d}z \end{aligned}}$$

$$\boxed{\tau_\nu = (1 + \eta_\nu)\tau}$$

Absorption & Scattering

Contribution to extinction coefficient due to absorption & scattering:

$$\boxed{\alpha_\nu = \alpha_\nu^a + \alpha_\nu^s}$$

Destruction probability = probability for absorption

$$\boxed{\varepsilon_\nu = \alpha_\nu^a \big/ (\alpha_\nu^a + \alpha_\nu^s)}$$

Scattering albedo = probability for photon being scattered

$$\boxed{\text{albedo}(\nu) = 1 - \varepsilon_\nu = \alpha_\nu^s \big/ (\alpha_\nu^a + \alpha_\nu^s)}$$

ERT: energy created and energy destroyed along beam:

$$\boxed{D = \underbrace{\varepsilon^c \rho \kappa^c I_\nu}_{\text{continuum absorption}} + \underbrace{(1-\varepsilon^c)\rho \kappa^c I_\nu}_{\text{continuum scattering}} + \underbrace{\varepsilon^l \rho \kappa_\nu^l I_\nu}_{\text{line absorption}} + \underbrace{(1-\varepsilon^l)\rho \kappa_\nu^l I_\nu}_{\text{line scattering}}}$$

$$\boxed{C = \underbrace{\varepsilon^c \rho \kappa^c B_\nu}_{\text{continuum emission}} + \underbrace{\varepsilon^l \rho \kappa_\nu^l B_\nu}_{\text{line emission}} + \underbrace{(1-\varepsilon^c)\rho \kappa^c J_\nu}_{\text{continuum scattering}} + \underbrace{(1-\varepsilon^l)\rho \kappa_\nu^l J_\nu}_{\text{line scattering}}}$$

ERT becomes:

$$\frac{\mu}{\rho}\frac{\mathrm{d}I_\nu(z)}{\mathrm{d}z} = (\varepsilon^c\kappa^c + \varepsilon_\nu^l\kappa_\nu^l)\,B_\nu(z) + [(1-\varepsilon^c)\kappa^c + (1-\varepsilon_\nu^l)\kappa_\nu^l]\,J_\nu(z) - (\kappa^c + \kappa_\nu^l)I_\nu(z)$$

Which simplifies to

$$\mu\frac{\mathrm{d}I_\nu(\tau_\nu)}{\mathrm{d}\tau_\nu} = I_\nu(\tau_\nu) - J_\nu(\tau_\nu) + \lambda_\nu[J_\nu(\tau_\nu) - B_\nu(\tau_\nu)]$$

where $$\lambda_\nu = \frac{\varepsilon^c\kappa^c + \varepsilon_\nu^l\kappa_\nu^l}{\kappa^c + \kappa_\nu^l} = \frac{\varepsilon^c + \varepsilon_\nu^l\eta_\nu}{1+\eta_\nu}$$

Eddington-Milne Solution

Assumptions to solve ERT above:

1. λ_ν independent of τ_ν (i.e., η_ν independent of τ)
2. $B_\nu(T[\tau])$ linear function of τ, continuum optical depth

   $B_\nu = a + b\,\tau = a + p_\nu\,\tau_\nu$

   $p_\nu = b\,/\,(1+\eta_\nu)$

Form moment equations:

$$\frac{\mathrm{d}H_\nu(\tau_\nu)}{\mathrm{d}\tau_\nu} = \lambda_\nu\,[J_\nu(\tau_\nu) - B_\nu(\tau_\nu)]$$

$$\frac{\mathrm{d}K_\nu(\tau_\nu)}{\mathrm{d}\tau_\nu} = H_\nu(\tau_\nu)$$

Use Eddington approximations $J = 3K$, to get from second moment equation above:

$$\frac{dJ_\nu(\tau_\nu)}{d\tau_\nu} = 3H_\nu(\tau_\nu)$$

Substitute this in first moment equation above and use linearity of B_ν to get:

$$\frac{d^2 J_\nu(\tau_\nu)}{d\tau_\nu^2} = 3\lambda_\nu\,[J_\nu(\tau_\nu) - B_\nu(\tau_\nu)]$$

$$\frac{d^2[J_\nu(\tau_\nu) - B_\nu(\tau_\nu)]}{d\tau_\nu^2} = 3\lambda_\nu\,[J_\nu(\tau_\nu) - B_\nu(\tau_\nu)]$$

Solution: $$J_\nu(\tau_\nu) - B_\nu(\tau_\nu) = C_1\,e^{-\sqrt{3\lambda_\nu}\tau_\nu} + C_2\,e^{+\sqrt{3\lambda_\nu}\tau_\nu}$$

Apply boundary conditions as before: no incident radiation, at large depth $J \Rightarrow B$, Eddington approximation, $J(0)=2H(0)$ or from exact solution $J(0)=3^{1/2}H(0)$

Using these approximations and boundary conditions gives

$$J_\nu = a + p_\nu\tau_\nu + \left(p_\nu - \sqrt{3}\,a\right)e^{-\sqrt{3\lambda_\nu}\tau_\nu} / \left[\sqrt{3} + \sqrt{3\lambda_\nu}\right]$$

and the emergent flux:

$$H_\nu(0) = J_\nu(0)/\sqrt{3} = \frac{1}{3}\left[p_\nu + \sqrt{3\lambda_\nu}\,a\right] / \left[1 + \sqrt{\lambda_\nu}\right]$$

For the continuum $\eta_\nu = 0$ so we get the residual flux as

$$R_\nu = H_\nu(0)/H^c(0) = \left[\frac{p_\nu + \sqrt{3\lambda_\nu}\,a}{1 + \sqrt{\lambda_\nu}}\right]\left[\frac{1 + \sqrt{\varepsilon^c}}{b + \sqrt{3\varepsilon^c}\,a}\right]$$