Interconnectedness

Moments (J_ν, H_ν, K_ν) depend on I_ν
Need to solve ERT to get I_ν
I_ν (and hence J_ν) depends on position and direction
I_ν depends on S_ν, hence on emissivity and opacity
Opacity depends on temperature and ionization
Temperature and ionization depends on J_ν

$$J_\nu = \frac{1}{4\pi}\int I_\nu \, \mathrm{d}\Omega$$

$$H_\nu = \frac{1}{4\pi}\int I_\nu \cos\theta \, \mathrm{d}\Omega$$

$$K_\nu = \frac{1}{4\pi}\int I_\nu \cos^2\theta \, \mathrm{d}\Omega$$

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = S_\nu - I_\nu$$

$$S_\nu \equiv \frac{j_\nu}{\alpha_\nu}$$

$$\mathrm{d}\tau_\nu = \alpha_\nu(s)\,\mathrm{d}s = \rho(s)\kappa_\nu\,\mathrm{d}s$$

Example: Model H II Region

- Sources of ionizing photons
- Opacity from neutral H: bound-free
- 1st iteration:
  - Medium fully ionized (no neutral H) so opacity is zero
  - Solve ERT throughout medium to get J_ν
  - Solve for ionization structure, some regions neutral
- 2nd iteration:
  - new opacity structure,
  - different solution for ERT, different J_ν values
  - new ionization and opacity structure
- Iterate until get convergence: solution of ERT, J_ν, ionization structure do not change with further iterations

Sources of Opacity

- Bound-bound transitions
- Bound-free transitions
- Free-free transitions (Bremstrahlung)
- Scattering: electrons, molecules, dust,…

Total opacity:

$$\rho\,\kappa(\nu) = \sum_i n_i\,\sigma_i(\nu)$$

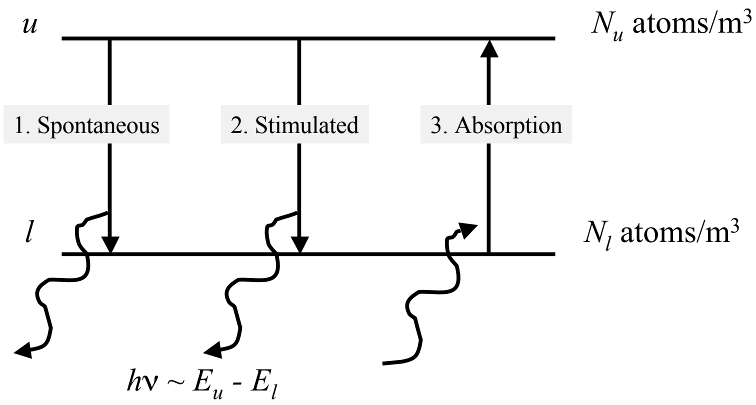
Bound-Bound Line Transitions

Between upper, *u,* and lower, *l,* levels of atom, ion, or molecule may occur as:

Spontaneous Radiative De-excitation
Radiative Excitation
Induced Radiative De-excitation
Collisional Excitation
Collisional De-excitation

Use *Einstein Coefficients* to describe these processes,

$$A_{ul}, B_{ul}, B_{lu}, C_{ul}, C_{lu}$$



Spontaneous De-excitation

$A_{ul} \equiv$ Probability for spontaneous de-excitation from u to l, per second, per particle in state u

Mean lifetime of particles in state u: $\Delta t = 1/A_{ul}$ secs
Spread in energy (Heisenberg) yields *Lorentz profile*:

$$\psi(\nu - \nu_0) = \frac{\gamma^{\text{rad}} / 4\pi^2}{(\nu - \nu_0)^2 + (\gamma^{\text{rad}} / 4\pi)^2}$$

$\gamma^{\text{rad}} = 1/\Delta t$
A_{ul} is a summation over the profile

Radiative Excitation

$B_{lu}\,\bar{J}^{\varphi}_{\nu_0} \equiv$ Number of radiative excitations from *l* to *u*, per second, per particle in state *l*

Induced De-excitation

$B_{ul}\,\bar{J}^{\chi}_{\nu_0} =$ Number of induced radiative de-excitations from *u* to *l*, per second, per particle in state *u*

Averaged mean intensity over line φ, χ:

$$\bar{J}^{\varphi}_{\nu_0} \equiv \int_0^\infty J_\nu \varphi(\nu-\nu_0)\,\mathrm{d}\nu\,; \quad \int_0^\infty \varphi(\nu-\nu_0)\,\mathrm{d}\nu = 1$$

Line Profiles

- Line profiles give probabilities for emitting/absorbing photon with frequency ν
- Number of atoms able to absorb photon = $N_l\ \varphi(\nu-\nu_0)$
- Complete redistribution: no "memory" of preceding processes: incident (exciting) and emitted photons not correlated
- Complete redistribution: profile shapes equal, $\psi = \varphi = \chi$

Collisional Excitation / De-excitation

$C_{lu} \equiv$ Number of collisional excitations from *l* to *u*, per second, per particle in state *l*

$C_{ul} \equiv$ Number of collisional de-excitations from *u* to *l*, per second, per particle in state *u*

Electron collisions usually most important:
Cooling in H II regions: collisions with electrons excite ions => de-excite and emit photons, carry away energy, hence important cooling source

Einstein Relations

Einstein coefficients coupled by the Einstein relations:

$$\frac{B_{lu}}{B_{ul}} = \frac{g_u}{g_l}; \qquad \frac{A_{ul}}{B_{ul}} = \frac{2h\nu^3}{c^2}$$

$$\frac{C_{lu}}{C_{ul}} = \frac{g_u}{g_l}\, \mathrm{e}^{E_{lu}/kT}$$

g = statistical weight or degeneracy: $g = 2J + 1$

Line Source Function

$$S_\nu^l \equiv j_\nu^l / \alpha_\nu^l = \frac{n_u A_{ul} \psi(\nu - \nu_0)}{n_l B_{lu} \varphi(\nu - \nu_0) - n_u B_{ul} \chi(\nu - \nu_0)}$$

Einstein relations:

$$S_\nu^l = \frac{\frac{A_{ul}}{B_{ul}} \frac{\psi}{\varphi}}{\frac{n_l}{n_u} \frac{B_{lu}}{B_{ul}} - \frac{\chi}{\varphi}} = \frac{2h\nu^3}{c^2} \frac{\psi / \varphi}{\frac{g_u n_l}{g_l n_u} - \frac{\chi}{\varphi}}$$

Complete Redistribution:

$$S_{\nu_0}^l = \frac{n_u A_{ul}}{n_l B_{lu} - n_u B_{ul}} = \frac{2h\nu_0^3}{c^2} \frac{1}{\frac{g_u n_l}{g_l n_u} - 1}$$

Continuum Transitions

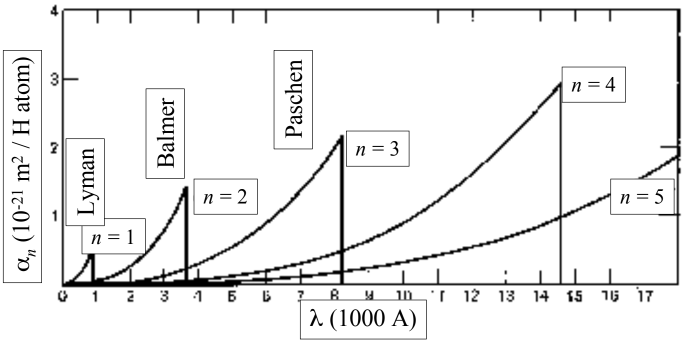
Bound-free:
Extinction cross section for hydrogen and hydrogen-like transitions, ***Kramer's*** formula:

$$\sigma_\nu^{\mathrm{bf}} = 2.815 \times 10^{25} \frac{Z^4}{n^5 \nu^3} g_{\mathrm{bf}} \ \mathrm{m}^2 \quad \text{for } \nu \geq \nu_0$$

n = principal quantum number of level *i* from which the atom or ion is ionized.
Z = ion charge and g_{bf} = *Gaunt factor*, QM correction ~ 1

Cross section decays as $1/\nu^3$ above the threshold ("edge") frequency ν_0 and is zero below it.

H I Continuous Extinction



e.g., Paschen continuum: absorptions to $n = 3$
Balmer Jump at $\lambda = 3646$ A
Peaks increase with n: Rydberg sequence $h\nu_n = 13.6/n^2$ eV

Free-free:

Thermal Bremsstrahlung, cross section is:

$$\sigma_\nu^{\mathrm{ff}} \approx 3.7 \times 10^4 N_{\mathrm{e}} N_{\mathrm{ion}} \frac{Z^2}{T^{1/2}\nu^3}\left(1 - \mathrm{e}^{-h\nu/kT}\right) g_{\mathrm{ff}} \ \mathrm{m}^2$$

Z = ion charge,
N_{e}, N_{ion} : electron, ion densities
g_{ff} = Gaunt correction factor

Scattering

Electron (Thomson) Scattering:
Frequency independent for low energy photons:

$$\sigma_\nu^{\mathrm{T}} \equiv \sigma^{\mathrm{T}} = \frac{8\pi}{3} r_{\mathrm{e}}^3 = 6.65 \times 10^{-29}\ \mathrm{m}^2$$

Frequency dependent:
High energy photons: ***Compton scattering***
High energy electrons: ***inverse-Compton scattering***

Thomson scattering is major source of continuous extinction in hot star atmospheres where H is ionized

Rayleigh Scattering:
Cross section for photons with $\nu << \nu_0$ by bound electrons with binding energy $h\,\nu_0$ is:

$$\sigma_\nu^{\mathrm{R}} \approx f_{lu} \sigma^{\mathrm{T}} \left(\frac{\nu}{\nu_0} \right)^4$$

f_{lu} and ν_0 characterize the major bound-bound “resonance transition” of the bound electron
e.g., Ly α transition in neutral hydrogen of a weighted sum over all Lyman lines
ν^4 $(1/\lambda^4)$ dependence makes the sky blue and sunsets red

Redistribution in Angle or Scattering Phase Function:

Thomson and Rayleigh scattering are ***coherent***: photon re-directed with same ν

Re-direction has ***phase function*** $\sim 1 + \cos^2\theta$

This is the angular shape for the scattered photons