Mass-luminosity relation

- **Use mass-radius relation and the expression for the constant C, again substituting for P_0 and T_0:**

$$C = \frac{3\kappa_0}{16\pi ac}\left(\frac{\mu m}{k}\right)^{1+\alpha}\left(\frac{GM^2}{4\pi R^4}\right)^{1+\alpha}\left(\frac{\mu m}{k}\frac{GM}{R}\right)^{-5-\alpha-\beta}\frac{L}{R}$$

$$\Rightarrow L \propto \frac{R^{3\alpha-\beta}}{M^{\alpha-3-\beta}}.$$

- **Use mass-radius relation to get L as function of M only, e.g. for α=0.5, β=2.5 and ν=16:**

$$L \propto \frac{M^5}{R} \text{ and } R \propto M^{13/18} \Rightarrow L \propto M^{5-13/18} \approx M^{4.3}.$$

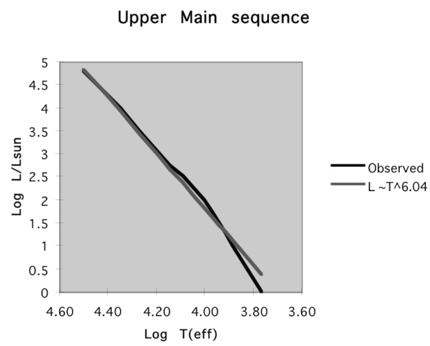
Slope of main sequence

- **Don't confuse internal temperature with effective (surface) temperature!**
- **Since** $T_{eff}^4 \propto \dfrac{L}{R^2}$,
- **Get for α=0.5 and β=2.5:**

$$T_{eff}^4 \propto \frac{M^{5-13/18}}{M^{26/18}} \Rightarrow T_{eff} \propto M^{0.71}$$

$$\Rightarrow L \propto T_{eff}^{(5-13/18)/0.71} = T_{eff}^{6.04}$$

- **Better fit to observed MS slope for early-type CNO burning stars than Kramers' Law (α=1, β=3.5)**



Heavy element abundance

- In deep interior, only heavy-element ions have enough electrons to contribute to b-f opacity.
- Hence κ_0 depends on heavy-element abundance.
- Note that ε_0 also depends on heavy element (^{14}N) abundances if CNO is dominant energy source, but not if p-p dominates.
- Consider stars of one solar mass or less powered by p-p chain (ν=4).
- Again eliminate L using product CD, including Z dependence of κ_0 and with α=0.5, β=2.5:

$$CD \propto Z \frac{\mu^{\nu-4-\beta} M^{\nu-1+\alpha-\beta}}{R^{\nu+3+3\alpha-\beta}} \propto Z \frac{M}{R^6}.$$

Different metallicity, constant mass

- At fixed mass this gives only a very weak dependence of radus on metallicity: $R \propto Z^{1/6}$
- Insert into equation for C at constant M:

$$C \propto Z \frac{\mu^{-4-\beta} M^{-3+\alpha-\beta}}{R^{3\alpha-\beta}} L \propto ZM^5 LR \propto ZLR \propto Z^{7/6} L$$

$$\Rightarrow L \propto Z^{-7/6}.$$

$$\text{and } T_{eff}^4 \propto \frac{L}{R^2} \propto \frac{1}{Z^{2/6} Z^{7/6}} \Rightarrow T_{eff} \propto Z^{-9/24}.$$

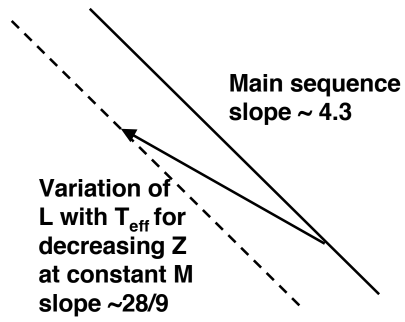
- Hence get L-T_{eff} relation:

$$Z \propto T_{eff}^{-24/9} \propto L^{-6/7} \Rightarrow L \propto T_{eff}^{28/9}.$$

- Shallower than slope of main sequence.

Metallicity and the main sequence

- **Main sequence moves blueward for low-metallicity systems.**
- **cf. location of globular-cluster main sequences in HR diagram.**
- **Also note that although metal poor "subdwarf"stars appear to lie below MS because they are less luminous at the same colour, they are really bluer and more luminous than metal-rich stars of the same mass!**



Beyond the ZAMS

- **So far we have taken no account of changes in composition as a star evolves.**
- **ZAMS star is initially well-mixed by convection.**
- **Subsequent H burning leads to**
  - **increase in molecular weight μ in core**
  - **decrease in core H abundance X.**
- **Again use CD with α=0.5, β=2.5 at constant mass to get:**

$$CD \propto Z \frac{\mu^{\nu-4-\beta} M^{\nu-1+\alpha-\beta}}{R^{\nu+3+3\alpha-\beta}} \Rightarrow \mu^{\nu-6.5} \propto R^{\nu+2}.$$

- **and get luminosity dependence from expression for C:**

$$C \propto Z \frac{\mu^{-4-\beta} M^{-3+\alpha-\beta}}{R^{3\alpha-\beta}} L \Rightarrow L \propto \mu^{6.5} R^{-1}$$

Why will the Sun become a red giant?

- **Helium accumulates in core.**
- **Core shrinks and heats up to maintain hydrostatic equilibrium as mol. wt. increases:**

$$kT_c = \left(\frac{\pi}{36}\right)^{1/3} G\overline{m}M^{2/3}\rho_c^{\ 1/3}.$$

- **Higher temperatures keep H burning going in thin shell surrounding He core, now dominated by CN cycle thanks to higher T.**
- **Luminosity and temperature are too great for low-mol. wt. material outside shell source.**
- **Excess internal pressure requires envelope to expand to maintain hydrostatic equilibrium.**
- **Shell source slowly moves out. Red giant expands further as He core mass grows.**

What causes the helium flash?

- **In stars with M < 2.25 M_{Sun} or so, helium core contracts until electrons become degenerate.**
- **Electron pressure now supports star, but temperature continues to increase.**
- **At core temp ~ 10^8 K, helium burning via 3α reaction begins.**
- **Energy released by He burning raises T further but has no effect on pressure: thermonuclear thermostat doesn't work.**
- **Thermal runaway raises temperature within seconds: brief but enormous increase in core luminosity "lifts" electron degeneracy.**
- **Star settles down in new equilibrium with expanded non-degenerate hot He-burning core.**

White dwarfs

- **Helium-burning phase ends with degenerate C-O core surrounded by layer of H, He.**
- **Electron degeneracy provides pressure support, so ρ is independent of T.**
- **If electrons are NR, use previous arguments to get central density needed for support:**

$$P = K_{\mathrm{NR}}\left[\frac{Y_e\rho_c}{m_{\mathrm{H}}}\right]^{5/3} \approx \left[\frac{\pi}{36}\right]^{1/3} GM^{2/3}\rho_c^{4/3}$$

$$\Rightarrow \rho_c \approx \frac{\pi}{36}\frac{G^3M^2}{K_{\mathrm{NR}}^3}\left[\frac{m_{\mathrm{H}}}{Y_e}\right]^5 \text{ where } Y_e \equiv \frac{n_e m_{\mathrm{H}}}{\rho_c}.$$

Number of electrons per nucleon (1 for pure H, ~ 0.5 for anything heavier)

The Chandrasekhar limit

- **Electrons at central density of a carbon WD with M = 0.4M_{Sun} have p_F~0.65 $m_e c$ and ε_F~0.19 $m_e c^2$.**
- **For more massive WD, approach equation of state for UR degenerate matter:**

$$P = K_{\mathrm{UR}}\left[\frac{Y_e\rho_c}{m_{\mathrm{H}}}\right]^{4/3} \approx \left[\frac{\pi}{36}\right]^{1/3} GM^{2/3}\rho_c^{4/3}$$

- **Central density cancels; get an expression for M (in the limit of infinite central density) in terms of Y_e and fundamental constants:**

$$M_{\mathrm{CH}} \approx \left[\frac{K_{\mathrm{UR}}}{G}\right]^{3/2}\left[\frac{Y_e}{m_{\mathrm{H}}}\right]^2\left[\frac{36}{\pi}\right]^{1/2} = 4.3Y_e^2 M_{\mathrm{Sun}}.$$

- **More accurate polytropic treatment gives M_{CH}~1.4M_{Sun} for Y_e ~0.5 (2 electrons per nucleon).**

Supernova collapse

- See AS2001 notes for details of advanced burning schemes leading to formation of ^{56}Fe core in stars with $M > 8M_{Sun}$.
- Contraction under gravity converts gravitational to internal energy:
  - if exothermic nuclear reactions result, internal KE and hence pressure increase, halting contraction BUT
  - if nuclear reactions result which ABSORB internal energy ("stellar refrigerator"), collapse proceeds unopposed.
- Nuclear photodisintegration:
  - $\gamma + {}^{56}Fe \rightarrow 13\,{}^{4}He + 4n$ absorbs $Q=(13m_4+4m_1-m_{56})c^2=124.4$ MeV
  - i.e. 1 kg of ^{56}Fe absorbs 2×10^{14} J
- Electron capture via inverse beta decay:
  - $e^- + {}^{56}Fe \rightarrow {}^{56}Mn + \nu_e$ -- neutrino escapes carrying energy.
  - threshold energy = $m_e c^2$+3.7 MeV.

Neutrino burst at core collapse

- Fermi energy of electrons in degenerate core equals E_{thresh} when $\rho_c=1.1\times10^{12}$ kg m^{-3}.
- Can also have $e^- + {}^{56}Mn \rightarrow {}^{56}Cr + \nu_e$
- At densities > 2×10^{13} kg m^{-3}, electron capture on to ^{56}Cr begins and proceeds rapidly.
- Average electron energy ~ 10 MeV at this point.
- Iron core with $M=M_{CH}$ contains ~10^{57} electrons which can produce 10^{57} neutrinos, carrying away 1.6×10^{45} J on free-fall timescale:

$$t_{\rm ff} = \left(\frac{3\pi}{32G\rho}\right)^{1/2} \sim 10^{-3}\ \text{s}.$$

- Neutrino mean free path ~ stellar radius (few km) at ρ~10^{14} kg m^{-3} – trapped for a few seconds before diffusing out of core.

THE END