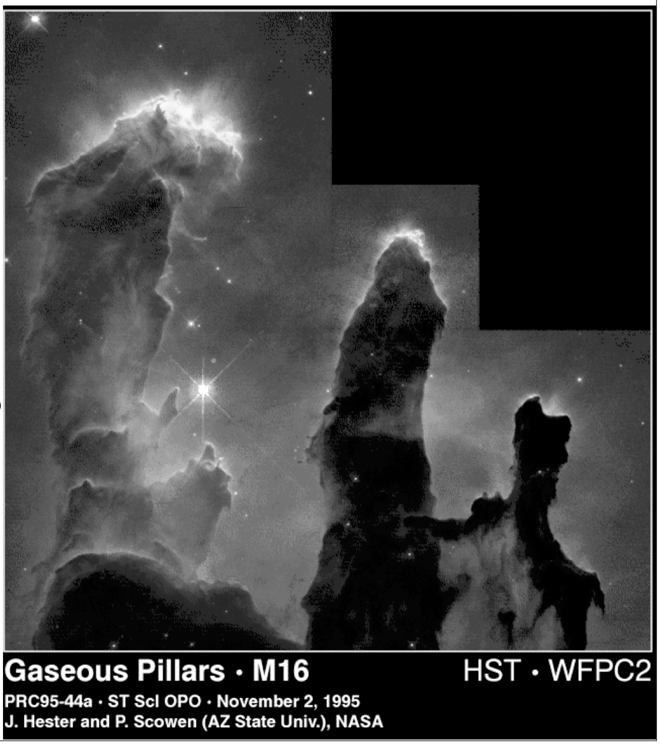
4. Star formation

- Galactic ISM: roughly uniform gas, n ~ 5×10^{6} m^{-3}.
- Stars form in molecular clouds.
- Dimensions ~ 10pc, density ~ 5×10^{9} m^{-3}, temperature ~ 10 K.
- Galactic magnetic field strongly tied to ionized plasma in ISM.
- Field lines run parallel to galactic plane.
- Local perturbations –> potential wells –> condensations.



4.1 Jeans' criterion

- Contracting cloud must be compact enough to ensure that dispersive effects of internal pressure don't overwhelm gravity.
- Energetically, cloud becomes bound if:

$$E_{\mathrm{grav}} + E_{\mathrm{kin}} < 0.$$

- Spherical cloud, mass M, radius R has gravitational binding energy:

$$E_{\mathrm{grav}} = \int_0^M \frac{Gm}{r}\, dm = -A \frac{GM^2}{R}$$

- where A depends on (and increases with) degree of central condensation of internal density distribution.
- A=3/5 for uniform density; we'll use A=1.

Conditions for collapse

- **Total thermal (kinetic) energy of cloud is:**

$$E_{\mathrm{kin}} = \frac{3}{2}\frac{M}{\mu m_{\mathrm{H}}}kT$$

Average mass of cloud particle

- **Critical condition: for collapse of a cloud with radius R to occur, need either:**

$$M_{\mathrm{cloud}} > M_{\mathrm{J}} = \frac{3kT}{2G\mu m_{\mathrm{H}}}R$$

Jeans mass

- **or**

$$\rho_{\mathrm{cloud}} > \rho_{\mathrm{J}} = \frac{3}{4\pi M^2}\left[\frac{3kT}{2G\mu m_{\mathrm{H}}}\right]^3$$

Jeans density

4.2 Onset of contraction

- **Contraction of a massive gas-dust cloud will proceed if not opposed by increasing internal pressure.**
- **Release of E_{grav} tends to increase internal temperature but also excites H_2 and other molecules into excited rotational levels.**
- **De-excitation emits photons mainly at IR and mm-wave frequencies where cloud is transparent.**
- **Hence photons escape, cooling the cloud and allowing contraction to proceed.**

4.3 Fragmentation and formation of protostars

- **As main cloud contracts, smaller subregions will also reach the Jeans density**
- **Fragments will contract independently provided gravitational PE is not converted to internal KE.**
- **Energy released can be absorbed by:**
  - **Dissociation of H_2 (ε_D = 4.5 eV)**
  - **ionisation of atomic H (ε_I = 13.6 eV)**
- **Amount of energy absorbed by this process is:**

$$\frac{M}{2m_{\mathrm{H}}}\varepsilon_{\mathrm{D}} + \frac{M}{m_{\mathrm{H}}}\varepsilon_{\mathrm{I}} .$$

Radius of a protostar

- **If we know the initial radius of the protostar we can calculate the final radius from:**

$$GM\left(\frac{1}{R_2} - \frac{1}{R_1}\right) \approx \frac{1}{m_{\mathrm{H}}}\left(\frac{\varepsilon_{\mathrm{D}}}{2} + \varepsilon_{\mathrm{I}}\right).$$

- **Example: Radius of a protostar of 1 solar mass is ~10^{15} m with a Jeans density ~10^{-16} kg m^{-3}.**
- **The dissociation + ionisation energy is 3×10^{39} J.**
- **The radius after gravitational contraction is R_2~ 10^{11} m.**
- **The timescale for this contraction is t_{dyn} ~ 20,000 y.**

4.4 Approach to hydrostatic equilibrium

- **After H is all ionised:**
  - **the internal pressure rises**
  - **contraction slows down**
  - **hydrostatic equilibrium is approached.**
- **Can use the Virial theorem to estimate the average internal temperature at this point.**
- **Total thermal KE of protons and electrons is:**

$$E_{\text{kin}} \sim \frac{3kT}{2}\frac{M}{\mu m_{\text{H}}} = \frac{3kTM}{m_{\text{H}}}.$$

- **Gravitational energy at end of collapse is:**

$$E_{\text{grav}} \sim -\frac{GM^2}{R_2} \sim -\frac{M}{m_{\text{H}}}\left(\frac{\varepsilon_{\text{D}}}{2} + \varepsilon_{\text{I}}\right).$$

4.5 Thermal contraction

- **Virial theorem: $2E_{kin}+E_{grav}=0$, so protostar approaches equilibrium at an average temperature**

$$kT \sim \frac{(\varepsilon_{\text{D}} + 2\varepsilon_{\text{I}})}{12} \sim 2.6 \text{ eV},$$

- **Corresponds to T~30,000 K.**
- **Independent of mass of protostar.**
- **Subsequent contraction governed by opacity, which controls loss of radiation from surface.**
- **Hence gravitational energy is radiated away on a thermal (Kelvin) timescale, t_K~10^7 – 10^8 y.**
- **Star remains close to hydrostatic equilibrium so we can continue to use Virial theorem.**

4.6 How far could contraction proceed without nuclear reactions?

- **Classical mechanics breaks down when wavefunctions of neighbouring electrons begin to overlap significantly, i.e. at separation**

$$r = \lambda_B = \frac{h}{m_e v}.$$

de Broglie wavelength

- **Since** $\frac{1}{2} m_e v^2 = \frac{3}{2} kT, \quad r = \frac{h}{(3 m_e kT)^{1/2}}$

$$\Rightarrow \rho = \frac{\mu m_H}{(4/3)\pi r^3}$$

$$\Rightarrow \rho \approx \mu m_H \frac{(m_e kT)^{3/2}}{h^3}.$$

Minimum mass of a star

- **At this point a further increase in density does not affect the temperature.**
- **Virial theorem gives average internal temperature:**

$$kT \approx \frac{GM\mu m_H}{3R} \approx G\mu m_H M^{2/3} \rho^{1/3}$$

- **Substitute to get maximum temperature:**

$$kT_{max} \approx \left[\frac{G^2 \mu m_H^{8/3} m_e}{h^2}\right] M^{4/3} \Rightarrow T_{max} \approx 10^7 \left(\frac{M}{M_{Sun}}\right)^{4/3}$$

- **For M < 0.08 M_{Sun}, T will not be high enough to trigger nuclear reactions.**