4. Star formation

- Galactic ISM: roughly uniform gas, \( n \sim 5 \times 10^6 \text{ m}^{-3} \).
- Stars form in molecular clouds.
- Dimensions \( \sim 10 \text{pc} \), density \( \sim 5 \times 10^9 \text{ m}^{-3} \), temperature \( \sim 10 \text{ K} \).
- Galactic magnetic field strongly tied to ionized plasma in ISM.
- Field lines run parallel to galactic plane.
- Local perturbations \( \rightarrow \) potential wells \( \rightarrow \) condensations.

4.1 Jeans’ criterion

- Contracting cloud must be compact enough to ensure that dispersive effects of internal pressure don’t overwhelm gravity.
- Energetically, cloud becomes bound if:
  \[ E_{\text{grav}} + E_{\text{kin}} < 0. \]
- Spherical cloud, mass \( M \), radius \( R \) has gravitational binding energy:
  \[ E_{\text{grav}} = \frac{GM}{r} \int_{0}^{M} dm = -A \frac{GM^2}{R} \]
  - where \( A \) depends on (and increases with) degree of central condensation of internal density distribution.
  - \( A=3/5 \) for uniform density; we’ll use \( A=1 \).
4.2 Onset of contraction

- Contraction of a massive gas-dust cloud will proceed if not opposed by increasing internal pressure.
- Release of $E_{\text{grav}}$ tends to increase internal temperature but also excites $H_2$ and other molecules into excited rotational levels.
- De-excitation emits photons mainly at IR and mm-wave frequencies where cloud is transparent.
- Hence photons escape, cooling the cloud and allowing contraction to proceed.
4.3 Fragmentation and formation of protostars

- As main cloud contracts, smaller subregions will also reach the Jeans density
- Fragments will contract independently provided gravitational PE is not converted to internal KE.
- Energy released can be absorbed by:
  - Dissociation of H$_2$ ($\varepsilon_D = 4.5$ eV)
  - Ionisation of atomic H ($\varepsilon_I = 13.6$ eV)
- Amount of energy absorbed by this process is:

\[
\frac{M}{2m_H} \varepsilon_D + \frac{M}{m_H} \varepsilon_I.
\]

Radius of a protostar

- If we know the initial radius of the protostar we can calculate the final radius from:

\[
GM \left( \frac{1}{R_2} - \frac{1}{R_1} \right) \approx \frac{1}{m_H} \left( \frac{\varepsilon_D}{2} + \varepsilon_I \right).
\]

- Example: Radius of a protostar of 1 solar mass is $\sim 10^{15}$ m with a Jeans density $\sim 10^{-16}$ kg m$^{-3}$.
- The dissociation + ionisation energy is $3 \times 10^{39}$ J.
- The radius after gravitational contraction is $R_2 \sim 10^{11}$ m.
- The timescale for this contraction is $t_{\text{dyn}} \sim 20,000$ y.
4.4 Approach to hydrostatic equilibrium

- After H is all ionised:
  - the internal pressure rises
  - contraction slows down
  - hydrostatic equilibrium is approached.
- Can use the Virial theorem to estimate the average internal temperature at this point.
- Total thermal KE of protons and electrons is:
  \[ E_{\text{kin}} \sim \frac{3kT}{2} \frac{M}{\mu m_H} = \frac{3kTM}{m_H}. \]
- Gravitational energy at end of collapse is:
  \[ E_{\text{grav}} \sim -\frac{GM^2}{R_2} \sim -\frac{M}{m_H} \left( \frac{\varepsilon_D}{2} + \varepsilon_1 \right). \]

4.5 Thermal contraction

- Virial theorem: \( 2E_{\text{kin}} + E_{\text{grav}} = 0 \), so protostar approaches equilibrium at an average temperature
  \[ kT \sim \frac{(\varepsilon_D + 2\varepsilon_1)}{12} \sim 2.6 \text{ eV}, \]
- Corresponds to \( T \sim 30,000 \) K.
- Independent of mass of protostar.
- Subsequent contraction governed by opacity, which controls loss of radiation from surface.
- Hence gravitational energy is radiated away on a thermal (Kelvin) timescale, \( t_K \sim 10^7 - 10^8 \) y.
- Star remains close to hydrostatic equilibrium so we can continue to use Virial theorem.
4.6 How far could contraction proceed without nuclear reactions?

- Classical mechanics breaks down when wavefunctions of neighbouring electrons begin to overlap significantly, i.e. at separation

\[ r = \frac{\lambda_B}{m_e v} \text{.} \]

- Since

\[ \frac{1}{2} m_e v^2 = \frac{3}{2} kT, \quad r = \frac{h}{(3m_e kT)^{1/2}} \]

\[ \Rightarrow \rho = \frac{\mu m_H}{(4/3)\pi r^3} \]

\[ \Rightarrow \rho = \frac{\mu m_H (m_e kT)^{3/2}}{h^3}. \]

Minimum mass of a star

- At this point a further increase in density does not affect the temperature.

- Virial theorem gives average internal temperature:

\[ kT \approx \frac{GM \mu m_H}{3R} = G \mu m_H M^{2/3} \rho^{1/3} \]

- Substitute to get maximum temperature:

\[ kT_{\text{max}} \approx \left[ \frac{G^2 \mu m_H^{8/3} m_e}{h^2} \right] M^{4/3} \Rightarrow T_{\text{max}} \approx 10^7 \left( \frac{M}{M_{\text{Sun}}} \right)^{4/3} \]

- For \( M < 0.08 M_{\text{Sun}} \), \( T \) will not be high enough to trigger nuclear reactions.