6. Stellar Opacity

- **Opacity κ appears in energy transport equation.**
- **Represents ability of stellar material to absorb radiation, or 1/(heat conductivity).**
- **Need to consider all microscopic processes that can absorb photons at each frequency ν:**
  - **Bound-bound (bb) absorption**
  - **Bound-free (bf) absorption**
  - **Free-free (ff) absorption**
  - **Electron scattering (es)**
- **Need to combine them all into a single macroscopic quantity.**
- **Also need to consider thermal (electron) conduction (degenerate white dwarf material).**

6.1 Opacity and mean free path

- **Efficiency of conductive processes depends on**
  - **density of carriers**
  - **energy per carrier**
  - **mean free path of carriers.**
- **For photons of energy E=hν, mean free path:**

$$\bar{l} = \frac{1}{\kappa_\nu \rho} \text{ where } \kappa_\nu = \kappa_{\nu,\mathrm{bb}} + \kappa_{\nu,\mathrm{bf}} + \kappa_{\nu,\mathrm{ff}} + \kappa_{\nu,\mathrm{es}}$$

bound-bound **bound-free** **free-free** **electron scattering**

6.2 Opacity in stellar interiors

- **Opacity depends on:**
  - **Level populations of different ion species (bb,bf)**
  - **Ionization balance (bf,ff)**
  - **Electron density ne (bf,ff,es)**
- **LTE is a good approximation inside stars.**
- **Level populations of individual species depend on T (Boltzmann) and on degree of ionization.**
- **Degree of ionization depends on n_e and T (Saha).**
- **Electron density n_e depends on ρ , T and local composition X_i. If fully ionized:**

$$n_{\mathrm{e}} \propto \rho(2X+Y) = \rho(1+X) \text{ since } X+Y \approx 1.$$

H contributes twice as many e^- per unit mass as He.

- **So ultimately $\kappa_\nu = \kappa_\nu(\rho,T,X_i)$.**
- **Need to define appropriate mean κ over all ν.**

6.3 Frequency-averaged opacity

- **How do we weight κ_ν when averaging over ν?**
- **Earlier we defined the *flux mean* opacity:**

$$\kappa \equiv \int_0^\infty \kappa_\nu F_\nu d\nu \Big/ \int_0^\infty F_\nu d\nu$$

- **Snag: have to solve the monochromatic equation of radiative transfer throughout the star to determine F_ν.**
- **We want a purely local approximation!**
- **Fortunately, in TE:**

$$P_{\mathrm{rad},\nu} = \frac{4\pi}{3c} B_\nu(T)$$

- **Hence**

$$-\frac{\kappa_\nu \rho}{c} F_\nu = \frac{dP_{\mathrm{rad},\nu}}{dr} = \frac{4\pi}{3c} \frac{dB_\nu(T)}{dT}\frac{dT}{dr}.$$

Chain rule!

Rosseland mean opacity - 1

- **Rearrange and integrate:**

$$\int_0^\infty F_\nu d\nu \left(= \frac{L}{4\pi r^2} \right) = -\frac{4\pi}{3\rho}\frac{dT}{dr}\int_0^\infty \frac{1}{\kappa_\nu}\frac{dB_\nu(T)}{dT}d\nu.$$

- **Define *Rosseland mean opacity* κ_R:**

$$\frac{1}{\kappa_R}\int_0^\infty \frac{dB_\nu(T)}{dT}d\nu \equiv \int_0^\infty \frac{1}{\kappa_\nu}\frac{dB_\nu(T)}{dT}d\nu.$$

- **Also:**

$$\int_0^\infty \frac{dB_\nu(T)}{dT}d\nu = \frac{d}{dT}\int_0^\infty B_\nu(T)d\nu = \frac{d}{dT}\left(\frac{acT^4}{4\pi}\right) = \frac{acT^3}{\pi}.$$

Rosseland mean opacity - 2

- **Rearrange to get radiative transport equation:**

$$\frac{dT}{dr} = -\frac{L}{4\pi r^2}\frac{3\kappa_R \rho}{4acT^3} \text{ where } \frac{1}{\kappa_R} \equiv \frac{\pi}{acT^3}\int_0^\infty \frac{1}{\kappa_\nu}\frac{dB_\nu(T)}{dT}d\nu.$$

- **i.e. same as before, but now we can calculate $\kappa_R = \kappa_R(\rho, T, X_i)$ from local conditions alone.**
- **Modern stellar structure calculations use tables of precalculated opacities for different chemical mixtures.**
- **Highly complex! Need to include all species and all bb, bf, ff and es processes.**
- **Opacity Project: data available online.**

Useful approximate opacities

- **Approximations useful in specific ranges of T and ρ for constructing simple stellar models:**
- **Intermediate T: bf and ff processes dominate:**

$$\kappa_{\rm bf} = \kappa^0_{\rm bf} Z(1+X)\rho T^{-7/2} \ {\rm m^2\ kg^{-1}},\ \kappa^0_{\rm bf} \approx 4\times 10^{24}$$

$$\kappa_{\rm ff} = \kappa^0_{\rm ff} (X+Y)(1+X)\rho T^{-7/2} \ {\rm m^2\ kg^{-1}},\ \kappa^0_{\rm ff} \approx 4\times 10^{21}$$

- **High T: electron scattering dominates:**

$$\kappa_{\rm es} = \frac{n_{\rm e}}{\rho}\alpha_{\rm es} = \frac{(1+X)}{2m_{\rm H}}\alpha_{\rm es} \approx 0.02(1+X)\ {\rm m^2\ kg^{-1}}.$$

$$\alpha_{\rm es} = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\varepsilon_0 mc^2}\right)^2$$

NB Check your AS2001 notes!

6.4 Diffusion approximation

- **Gradient in energy density of carriers:**

$$\frac{du}{dr} = \frac{du}{dT}\frac{dT}{dr} = C\frac{dT}{dr}.$$

C=Heat capacity

- **Heat flux density is then:**

$$F(r) = -K\frac{dT}{dr},\ \text{where } K \approx \frac{1}{3}\bar{v}\,\bar{l}\,C$$

Thermal diffusivity (K) — **Mean speed** ($\bar{v}$) — **Mean free path** ($\bar{l}$)

- **For photons:**

$$u_{\rm r} = aT^4,\quad C_{\rm r} = 4aT^3 \Rightarrow \quad K_{\rm r} \approx \frac{4acT^3}{3\kappa\rho}.$$

- **For electrons:**

$$u_{\rm e} = \frac{3}{2}n_{\rm e}kT,\quad C_{\rm e} = \frac{3}{2}n_{\rm e}k,\quad \bar{v}_{\rm e} = \left(\frac{3kT}{m_{\rm e}}\right)^{1/2}.$$

6.5 Electron conduction

- **e^- - ion collisions transfer energy more efficiently than e^- - e^- collisions.**
- **Mean free path for e^- to collide with ion is:**

$$\bar{l} = \frac{1}{n_i \sigma};\ \text{estimate } \sigma = \pi r^2$$

- **where r is the distance at which potential energy of e^- - ion pair matches thermal KE:**

$$\frac{Ze^2}{4\pi\varepsilon_0 r} \approx kT \Rightarrow K_e \approx \frac{k}{2\pi}\left(\frac{3kT}{m_e}\right)^{1/2}\left(\frac{4\pi\varepsilon_0 kT}{Ze^2}\right)^2$$

$$\text{Total opacity}: 1/\kappa_{tot} = 1/\kappa_{rad} + 1/\kappa_{cond},$$

$$\text{where } \kappa_{cond} = \frac{4acT^3}{3K_e \rho}.$$

From diffusion approximation

Electron vs photon conduction

- **Use electron scattering cross-section to get a lower limit on photon conductivity:**

$$\bar{l} = \frac{1}{n_e \alpha_{es}};\ \text{if we assume that } n_e \approx Z n_i,$$

$$\text{get } \frac{K_r}{K_e} \approx \frac{Z}{\sqrt{3}} \frac{aT^3}{n_e k}\left(\frac{m_e c^2}{kT}\right)^{5/2} = \sqrt{3} Z \frac{P_r}{P_e}\left(\frac{m_e c^2}{kT}\right)^{5/2}$$

- **e.g. hydrogen plasma, solar interior conditions:**

$$T = 10^6\,\text{K},\ \rho = 1.4 \times 10^3\,\text{kg m}^{-3}$$

$$\Rightarrow kT \approx 10^{-3} m_e c^2, P_r = 3 \times 10^{11}\,\text{Pa}, P_e = 7 \times 10^{13}\,\text{Pa}$$

$$\Rightarrow K_r = 2 \times 10^5 K_e,\ \text{i.e. photons are more effective.}$$

Electron conduction in white dwarfs

- **Photon mean free path becomes short in dense material.**
- **Electrons form dense degenerate gas with high conductivity, cf.metals.**
- **If Fermi energy ε_F >> kT:**

  $\bar{v}_e$ increases by factor $\sim (\varepsilon_F/kT)^{1/2}$,

  C_e decreases by factor $\sim kT/\varepsilon_F$.
- **Mean free path is also longer: e^- only scattered if unoccupied state available to be filled.**
- **High electron conductivity gives nearly uniform temperature throughout degenerate interior.**
- **Surrounded by insulating outer layer of non-degenerate e^- and ions.**