Lecture 11: The luminosity distribution of galaxies (cont'd)

- How many galaxies are there ?
- How do we calculate this ?
- How do we represent it ?
- What's the implication for:
  - The luminosity density
  - The matter density
- Some example calculations

The Schechter function

To represent the luminosity distribution, we use the Schechter function (Schechter 1976):

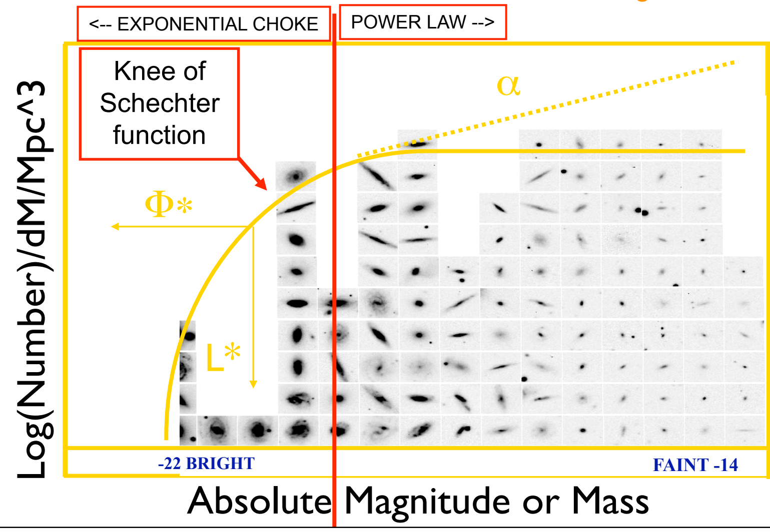
Faint-end slope parameter, α

$$\Phi(L)dL = \Phi_*\left(\frac{L}{L_*}\right)^{\alpha} e^{-(L/L_*)}\,dL$$

Normalisation Point, Φ_*

Characteristic Luminosity, L_*

- Schechter fn (1976) developed from Press Schechter theory
- Essentially a Gamma function (power law + exponential)
- Directly yields luminosity and mass density (i.e., Omegas)
- A foundation measurement vital for all of extragalactic astronomy



The Schechter function

- Arbitrarily based on observation that at low intrinsic luminosities Φ(L) behaves as a power law and that this power law is truncated at bright intrinsic luminosities, I.e., very few very bright galaxies are seen:

$$\Phi(L)dL \propto \left(\frac{L}{L_*}\right)^{+\alpha} e^{-(L/L_*)} dL$$

POWER LAW

EXPONENTIAL CHOKE

For: $L >> L_*,\ e^{-(L/L_*)} \rightarrow 0$

$L >> L_*,\ e^{-(L/L_*)} \rightarrow 1$

$L >> L_*,\ e^{-(L/L_*)} \approx \left(\frac{L}{L_*}\right)^{+\alpha}$

[For reasonable α]

- But, we typically work in magnitudes so lets convert L--->M and dL--->dM:

$$\Phi(L)dL = \phi_*(\frac{L}{L_*})^{+\alpha} e^{-(L/L_*)} dL$$

$$\frac{L}{L_*} = 10^{-0.4(M-M_*)} \qquad [1]$$

$$\frac{dL}{dM} = \frac{\delta(10^{-0.4(M-M_*)})}{\delta M} = \frac{\delta(e^{\ln[10^{0.4(M_*-M)}]})}{\delta M} = \frac{\delta(e^{0.4(M_*-M)\ln 10})}{\delta M}$$

$$\frac{dL}{dM} = 0.4\ln 10 e^{0.4(M_*-M)\ln 10}$$

$$dL = (0.4\ln 10).10^{0.4(M_*-M)} dM \qquad [2]$$

- Substituting [1] & [2] into Schechter fn:

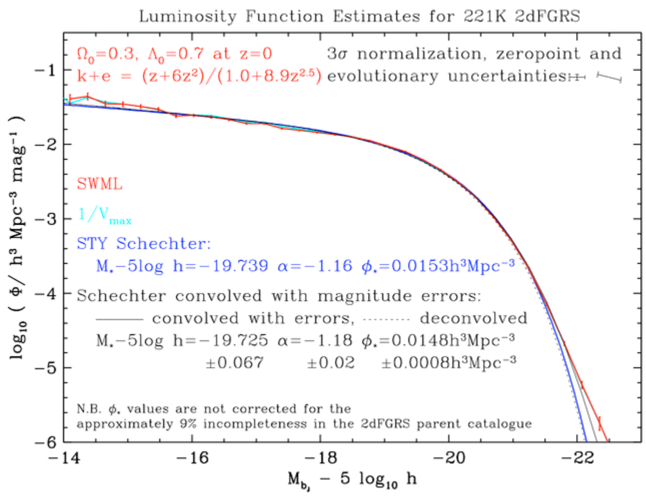
$$\Phi(M)dM = 0.4\ln 10\phi_* 10^{0.4(M_*-M)} 10^{0.4(M_*-M)\alpha} e^{-10^{0.4(M_*-M)}} dM$$

$$\Phi(M)dM = 0.4\ln 10\phi_* 10^{0.4(M_*-M)(\alpha+1)} e^{-10^{0.4(M_*-M)}} dM$$

- Hence with three parameters we can define the space density of galaxies:
  - ϕ_* = Normalisation of the luminosity function
  - α = Faint-end slope parameter
  - M_* = Charachterstic absolute magnitude at normalisation point

- Typical values (and units):
  - ϕ_*=0.002$(h_{0.5})^3$Mpc^{-3} +/- 10%
  - α= -1.20 +/- 0.1
  - M_*=-21.0+5log$(h_{0.5})$ +/- 0.25 mag

- this is the Schechter function derived from the 2DF survey



- Note dependence on H_0:

$$h_{0.5} = \frac{H_0}{50} = 1 \text{ if } H_0 = 50 km/s/Mpc$$

$$h_{1.0} = \frac{H_0}{100} = 1 \text{ if } H_0 = 100 km/s/Mpc$$

- Sometimes see $h_{0.75}$ or $h_{0.68}$,why ?
- Because z~v/c & d=v/H_0
- So converting from z to distance requires knowing H_0
- I.e.,

$$m - M = 5\log d + 25$$
$$m - M = 5\log(cz/H_0) + 25$$
$$m - M = 5\log(cz/H_0) + 25$$
$$m - M = 5\log(cz/50h_{0.5}) + 25$$
$$m - M = 5\log(cz/50) - 5\log h_{0.5} + 25$$
$$M = m - 5\log(cz/50) - 25 + \underline{5\log h_{0.5}}$$

$$\phi_* \propto V^{-1} \propto d^{-3} \propto H_0^3$$
$$\therefore \phi_* = N\underline{h_{0.5}^3} Mpc^{-3}$$

E.g., convert previous values of ϕ_*, α, M_* from H_o=50 to H_o=68 km/s/Mpc ?

$$h_{0.5} = \frac{H_0}{50} = \frac{68}{50} = 1.36$$
$$\phi_* = 0.002 h_{0.5}^3 = 0.002(1.36)^3 = 5.03 \times 10^{-3} Mpc^{-3}$$
$$\alpha = -1.20$$
$$M_* = -21 + 5\log h_{0.5} = -20.3\text{mag}$$

Luminosity and mass density of U

- From here we can now measure the average luminosity density of the local universe and then by adopting an average mass-to-light ratio we can get to the average local mass density.
- Here's how:
- Lets define J to be the luminosity density per Mpc^3

$$\therefore J = \int_0^\infty L\Phi(L)dL = \int_0^\infty L_* \frac{L}{L_*}\Phi(L)dL$$
$$J = \phi_* L_* \int_0^\infty (\frac{L}{L_*})^{\alpha+1} e^{-(L/L_*)} dL = \phi_* L_* \Gamma(\alpha+2)$$

Euler's integral, i.e., Gamma fn

- Note if: $\alpha \le -2, J \uparrow \infty \therefore \alpha \ge -2$
- For integer α:

$$J = \phi_* L_* (\alpha+1)!$$

(H_o dependant)

Density of Universe ?

- Mean density of universe = J(M/L) (M/L=mass-to-light ratio)

$$\bar{\rho} = J\frac{\overline{M}}{L} = \frac{\overline{M}}{L}\phi_* L_* \Gamma(\alpha+2)$$

$If:$

$\alpha = -1$

$L_* = 10^{10} L_{\oplus} h^{-2}$

$\phi_* = 0.002 h^3 Mpc^{-3}$

$\frac{\overline{M}}{L} = 10\frac{M_{\oplus}}{L_{\oplus}} h$

$M_{\oplus} = 2\times 10^{30} kg$

$1Mpc = 3\times 10^{22} m$

$$\therefore \bar{\rho} = \frac{\overline{M}}{L}\phi_* L_* (\alpha+1)! = 10\frac{M_{\oplus}}{L_{\oplus}} h_{0.5} 0.002 h_{0.5}^3 10^{10} L_{\oplus} h_{0.5}^{-2} \frac{1}{(3\times 10^{22})^3}$$

$$\bar{\rho} = 1.48\times 10^{-29} h_{0.5}^2 \mathrm{kgm}^{-3}$$

Q) From cosmology a density (today) of: $\rho_o = \frac{3H_0^2}{8\pi G}$ is required to close the universe, Based on the previous calculation and that $G=6.67\times 10^{-11} kg^{-1} m^3 s^{-1}$ is our Universe open or close ?

What M/L is required to just close the universe ?

$$\rho_0 = \frac{3[\frac{50\times 10^3 h_{0.5}}{3\times 10^{22}}]^2}{8\pi 6.67\times 10^{-11}} = 5\times 10^{-27} h_{0.5}^2 \mathrm{kg\ m}^{-3}$$

$$\bar{\rho} << \rho_o, \ \therefore \text{Universe is open}$$

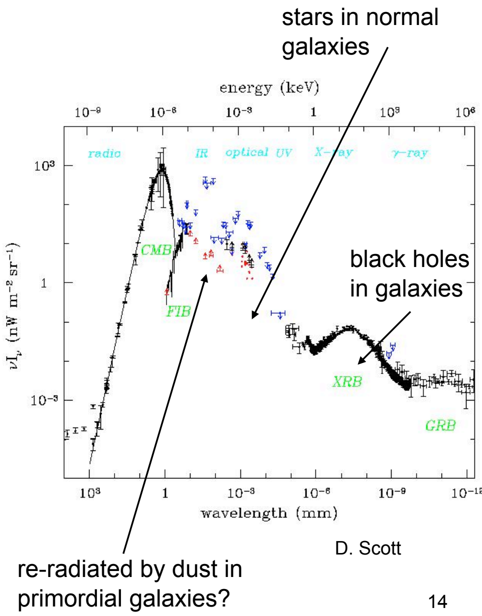
$$\frac{\mathrm{M}}{\mathrm{L}} \text{ required } = 10\frac{\rho_0}{\bar{\rho}} = 3359\frac{M_{\oplus}}{L_{\oplus}}!$$

other matter

- so where's the rest of the matter?
- we can estimate the baryon density needed to make the elements that were products after the Big Bang
  - H, D, He, a bit of Li
  - this gives $\rho_B \sim 4 \times 10^9\ M_{solar}\ Mpc^{-3}$
- and we have already worked out the total light density
  - evaluated as $J \sim 1.5 \times 10^8\ h\ L_{solar}\ Mpc^{-3}$
- so dividing these, the *baryonic* M/L ratio of the Universe has to be around $25\ h^{-1}\ M_{solar}/L_{solar}$
  - considerably more than the values for stars, so there is baryonic matter that we haven't detected

other light

- there are also radiation backgrounds that we haven't considered yet
  - plot shows energy densities, so integrate over wavelength to get total energy
  - the cosmic microwave background has the most energy
- others are less understood
  - partly from individual sources, partly unresolved



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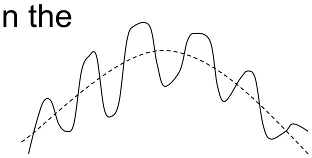
formation of galaxies

- probably the largest galaxies formed first out of the most over-dense regions in the young Universe
  - imagine small ripples superimposed on larger ones
  - parts sticking up the most form the first galaxies
  - large scale structure develops later



- maps of the CMB show huge structures
  - e.g. the COBE map, regions 7° across enclose 10^{20} M_{solar}, with ripples only ~1 part in 1000
  - these are *not* galaxy seeds